# Lecture 4: Washer method and work

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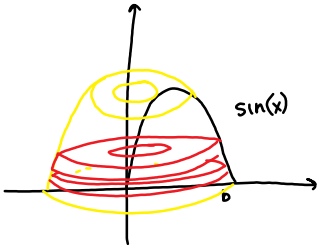
## Volumes using integrals while rotating around the y-axis (washer method)




Volume in these situations is given by:

$$V = \int_0^b 2\pi x\, f(x)\, dx$$

This is called the **washer method**.

Ex Volume of $y = 2x^2 - x^3$, rotated around the y-axis

(1) Start at y=0 (baseline is the x-axis)

(2) Find intersection points of y with the x-axis

$0 = 2x^2 - x^3$

$0 = x^2(2 - x)$

$x = 0, x = 2$

(3) Calculate the volume

$$V = \int_0^2 (2\pi x)\left(2x^2 - x^3\right) dx$$

$$= \int_0^2 \left(4\pi x^3 - 2\pi x^4\right) dx$$

$$= \pi\left(x^4 - \frac{2}{5}x^5\right)\Big|_0^2$$

$$= \frac{16}{5}\pi$$

## Work (6.4)

2nd derivative of displacement (acceleration)

Recall: Newton's law of motion

(1) $F = m \cdot g = m \cdot \frac{d^2s}{dt^2}$, unit = N(ewtons)

(2) $W = F \cdot s$, unit = J(oules)

work force displacement

Ex How much work is done when biking up a 350m hill? The cyclist weighs 55kg, the bike weighs 11kg, and the water and tools you're carrying weigh 2kg.

$s = 350m$

$g = 9.81\frac{m}{s^2}$

$F = (55 + 11 + 2)(9.81)$

$= 667.08N$

$W = F \cdot s$

$= 667.08N \cdot 350m$

$= 2334783\, J$

Ex A force of 40N is required to hold a spring stretched from its natural length of 10cm to 15cm. How much work is required to stretch the spring 3 more cm (from 15cm to 18cm)?

Recall:

**Hooke's Law**: $F = kx$

First, compute $k$:

$F = k\Delta x$

$40 = k(0.15 - 0.1)$

$k = 800$

So, a function of force with respect to $x$ is:

$f(x) = 800x$

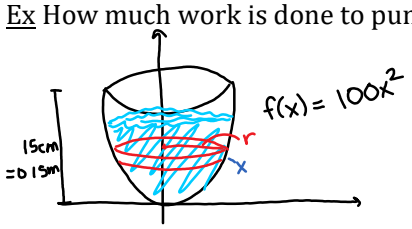
To calculate the work done, the force function is no longer constant so we need to use integrals.

$$W = \int_{0.05}^{0.08} 800x \; dx$$ bounds start 5cm *beyond* natural length of 10cm

$$= 400\left(0.08^2\right) - 400(0.05)^2$$

$$= 1.56\, J$$

Ex How much work is done to pump the water out of the solid below a) from the top and b) from the bottom?




We need to lift water $x$ cm where $x$ varies as the water surface gets lower and lower

1. Find cross section surface area:
   - need radius $r$ of the disk:

     $y = 100x^2$, where $x$ is the radius

     $x = \frac{\sqrt{y}}{10}$, this is the radius at height $y$!

Calculate the work done

$$W = \underbrace{d}_{\text{mass of water}} \cdot g \cdot \int_0^{0.15} \underbrace{\frac{y}{100}\pi}_{\text{disc area of radius } x \text{ @ height } y} \underbrace{(0.15 - y)}_{\text{distance to top from height } y,\ \text{ie how much we need to lift water}} dy$$

$$= \frac{dg\pi}{100} \int_0^{0.15} \left(0.15y - y^2\right) dy$$

$= \ldots$

$= 0.173\, J$